On-the-job search, mismatch and efficiency*

Pieter A. Gautier[†] **Coen N. Teulings** [‡] **Aico van Vuuren**[§]

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Abstract

This paper characterizes the equilibrium for a large class of search models with two-sided heterogeneity and on-the-job search. Besides the well-known congestion externalities, we show that on-the-job search in combination with monopsonistic wage setting *without* commitment creates a "business-stealing" externality. In the absence of congestion effects, this leads to excessive vacancy creation. Under wage setting *with* commitment this externality is absent because when posting a wage, firms take into account the expected productivity of future workers in their current jobs. If firms are able to make and respond to counteroffers, then they will not have to pay no-quit premia and this also leads to excessive vacancy creation.

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[†]Vrije Universiteit Amsterdam, Tinbergen Institute, CEPR, IZA. *Address:* Department of Economics, Free University Amsterdam, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands. email: pgautier@feweb.vu.nl

[‡]CPB Netherlands Bureau for Economic Policy Analysis, University of Amsterdam, Tinbergen Institute, CEPR, CES-IFO, IZA *Address:* Van Stolkweg 14, NL-2585 JR The Hague, The Netherlands. email: C.N.Teulings@cpb.nl

[§]Vrije Universiteit Amsterdam, *Address:* Department of Economics, Free University Amsterdam, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands. email: avuuren@feweb.vu.nl.

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1 Introduction

Two out of every three jobs taken by young workers end within a year and many of those separations reflect job-to-job changes rather than layoffs (see Topel and Ward, 1992).[1] This suggests that there are search or information frictions in the labor market that prevent worker types from immediately matching with their optimal job type. Any model that aims to adequately describe actual labor market flows should therefore allow for job-to-job transitions, which is the aim of this paper. Specifically, we consider an assignment model with search frictions, on-the-job search, two-sided heterogeneity (both for workers and jobs) and free entry of vacancies. Incorporating heterogeneity into a search model is relevant because one of the most important reasons for search is to find "*the right man for the job*". Search frictions frustrate this process and make the assignment of workers to jobs imperfect (see Teulings and Gautier, 2004) for an analysis without on-the-job search. They use an hierarchical model in which better-skilled workers have a comparative advantage in more complex jobs. Hierarchical models however, are hard to analyze. This paper therefore uses a circular model similar to Marimon and Zilibotti (1999).[2]

This paper analyzes the social efficiency of various wage-setting mechanisms, including wage posting with commitment (as in Burdett and Mortensen,1998) and monopsony without commitment of firms on future wage payments (which is in the same spirit as Shimer, 2006, except that we assume that firms have all the bargaining power). Allowing for heterogeneous matches makes it possible to compare the effects of commitment and no commitment in a unified framework. We show for the no-commitment case that when off- and on-the-job search are equally efficient, firms tend to create too many vacancies due to a *business-stealing effect*. When opening a vacancy, firms take into account that a higher wage makes it less likely that their worker quits, but they do not internalize the output losses that a job switcher imposes on her previous employer. The business-stealing externality is absent under wage posting however, because in addition to the no-quit premium, firms pay a hiring premium, which exactly offsets the business-stealing externality.

[1] Only for workers with less than a year of labor market experience the lay-off rate is larger than the job-to-job transition rate. Fallick and Fleischman (2004) and Nagypál (2005) also give evidence on the importance of on-the-job search. They show that in the US, the rate of job-to-job transitions is twice as high as the rate at which workers move from employment to unemployment.

[2] The results of Gautier, Teulings and Van Vuuren (2005) show that without on-the-job search, the circular model has the same characteristics as a Taylor expansion of the hierarchical model.

We show that for a given worker type, the no-quit premium equals the hiring premium because the value of a match for a given firm depends only on the worker type and not on her previous state. Finally, we show that the wage mechanism of Postel-Vinay and Robin (2002) is also inefficient because firms can appropriate all of the rents of job matches in the future. The possibility of making counteroffers eliminates the need to pay no-quit premia, and both hiring- and no-quit premia are necessary for a socially efficient outcome.

This paper is related to a number of other papers. Pissarides (1994) also studies on-the-job search in a matching framework. His model differs from ours in at least three ways: (1) he considers identical workers and two job types, while we consider a continuum of different workers and jobs, (2) our model generalizes the contact technology and allows congestion externalities to be switched on or off, (3) in our model, wages are determined as in Shimer (2006) or by wage posting, while Pissarides assumes a linear sharing rule. In Barlevy (2002), wages are also determined by a linear sharing rule, and he uses a different contact technology. His focus also differs from ours: he makes the important point that the sullying effect of recessions (workers move more slowly to their optimal job types because of low vacancy creation in recessions) dominates the positive cleansing effect of recessions for realistic parameter values, while we focus on efficiency. As in Jovanovic (1979, 1984), the model in this paper predicts individual separation probabilities to be decreasing in job tenure because the good matches are the ones that survive. Burdett and Mortensen (1998) show that worker- and job heterogeneity are not necessary to explain job-to-job movements. Finally, whereas on-the-job search with two-sided heterogeneity is studied in Mortensen (2000), Bontemps, Robin and Van den Berg (2000), Gautier (2002) and Moscarini (2005), these papers do not discuss efficiency issues. We show that the wage mechanisms in those papers are typically inefficient because of the absence of hiring- and/or no-quit premia.

The paper is organized as follows. Section 2 starts with the assumptions and derives the equilibrium conditions. Section 3 characterizes the equilibrium. Section 4 conducts welfare analysis and Section 5 concludes.

2 The model

The economy that is considered in this paper has the following properties.

Production

There is a continuum of worker types s and job types c; s and c are locations on a circle with circumference 1, so that $s = 1$ is equivalent to $s = 0$, and the same for c. Workers can only produce output when matched to a job. The productivity Y of a match depends only on the "distance" between s and c: $x(s,c) = \min_{k\in\mathbb{Z}} |s - c + k|$. Hence: $Y = Y(x)$. We assume that $Y(x)$ is twice differentiable, strictly quasi concave and has an interior maximum. Without loss of generality, this maximum is located at $x = 0$, and the value of the maximum is normalized to unity: $Y(0) = 1$. Hence, x is a measure of the degree of mismatch of an assignment. These assumptions imply that $Y_x(0) = 0$, since $x = 0$ maximizes $Y(x)$. We consider one of the simplest functional forms that meets those criteria:

$$Y(x) = 1 - \frac{1}{2}\gamma x^2.$$

This functional form can be interpreted as a second-order Taylor series expansion of a more general production function. We are interested in non-trivial equilibria where unemployed workers do not accept all jobs.[3] The parameter γ is related to the complexity dispersion parameter discussed in Teulings and Gautier (2004: 558) and Teulings (2005). Low values of γ imply that worker types are close substitutes.

Labor supply

We assume that labor supply per s-type is uniformly distributed over the circumference of the circle and that total labor supply in period t equals $L(t)$. Unemployed workers receive a value of leisure B. Employed workers supply a fixed amount of labor, and their payoff is equal to the wage they receive.

Labor demand

There is free entry of vacancies for all c-types. The flow cost of maintaining a vacancy is equal to K per period. After a vacancy is filled, the firm only pays for the wage of the worker.

Golden-growth path

We study the economy while it is on a golden-growth path, where the discount rate $\rho > 0$ approaches the growth rate of the labor force. We normalize the labor force at $t = 0$ to one, which implies that the size of the labor force at time t, $L(t)$, equals $\exp(\rho t)$. This

[3] This requires $Y(x) < 0$ for at least some x. Since $0 \leq x \leq \frac{1}{2}$, a sufficient condition for an equilibrium where not all jobs are accepted is $\gamma > 8$. When $\gamma = 0$, workers and firms are identical.

assumption buys us quite a lot in terms of transparency and tractability. In our analysis we focus on the balanced growth path: *i.e.* u and v are independent of time. We assume that all new workers start out as unemployed. The implications of the golden-growth assumptions are equivalent to those that follow from assuming that the discount rate ρ goes to zero, or more precisely, that the discount rate is much smaller than the job-finding and separation rate, $\rho \ll \delta, \lambda$. This is a common assumption in the wage-posting literature (see for example, Burdett and Mortensen,1998).

Job-search technology

Let m be the total number of contacts between job seekers and vacancies per unit of labor supply, u is the unemployment rate, and v is the total number of vacancies per unit of labor supply. Then, the general expression for the contact technology between job seekers and vacancies reads as follows:

$$\begin{aligned} mL(t) &= \mu(t) \left\{ \left[u + \psi \left(1 - u \right) \right] L(t) \right\}^{\eta} \left\{ vL(t) \right\}^{\xi} \\ m &= \lambda(t) \left[u + \psi \left(1 - u \right) \right]^{\eta} v^{\xi} \\ \lambda(t) &= \mu(t) L(t)^{\eta + \xi - 1} \end{aligned}$$

The parameter $\psi, 0 \leq \psi \leq 1$, measures the efficiency of on-the-job search relative to search while unemployed; $\psi = 0$ is the case with no on-the-job search, as analyzed in the stochastic matching model of Pissarides (2000), Marimon and Zilibotti (1999) and Teulings and Gautier (2004).[4] $\psi = 1$ is the case where off- and on-the-job search are equally efficient. The parameters $\eta, 0 < \eta \leq 1$ and $\xi, 0 < \xi \leq 1$ measure the relative contributions of job seekers and vacancies to the contact process; $\eta = \xi$ is the symmetric case where both inputs yield an equal contribution; $\eta + \xi$ measures the returns to scale of the contact process; $\eta + \xi = 1$ yields the standard matching function with constant returns to scale (CRS), a higher value, $\eta + \xi > 1$ yields increasing returns (IRS). The case $\eta = \xi = 1$ is the quadratic contact technology that has also been frequently used in the search literature. Hence, our specification of the contact technology encompasses the quadratic and CRS technology as special cases. Teulings and Gautier (2004) provide a number of motivations to understand why the quadratic technology might be the most adequate assumption in a model with two-sided heterogeneity the main reason being that this technology avoids congestion effects between different worker- and job types. The parameter $\mu(t)$ (and hence $\lambda(t)$) measures the overall efficiency of the matching process.

[4] We refer to the working paper version of this paper for a full analysis of this case.

The limiting case $\lambda(t) \to \infty$ yields the Walrasian equilibrium. For the non-CRS case, the parameter $\lambda(t)$ also captures the scale of the labor market. Under a quadratic contact technology (which exhibits IRS), the growth of the labor force implies an upward trend in the efficiency of the search process. For the sake of simplicity, we assume that there is an offsetting downward trend in the efficiency of the market so that $\lambda(t)$ remains constant and the labor market does not continuously become more efficient over time. Hence, for the remaining analysis we drop the argument of λ.[5] The contact technology is defined for a labor supply normalized to one. A 1% increase in labor supply for the same number of vacancies per unit of labor supply v would raise the number of contacts per unit of labor supply, m, by $\eta + \xi - 1\%$. This effect is assumed to be captured by λ. Hence, for IRS, a Walrasian labor market is equivalent to a labor market of infinite size, since both imply $\lambda \to \infty$.

Let $S(u, v) \equiv [u + \psi(1 - u)]^{\eta - 1} v^{\xi - 1}$. The implied contact rates for (un)employed workers and vacancies are

$$\begin{aligned} \lambda_{(s,\text{unemployed})\to\text{job}} &= \lambda v S, \\ \lambda_{(s,\text{employed in } z)\to\text{job}} &= \psi \lambda v S, \\ \lambda_{c\to\text{unemployed}} &= \lambda u S, \\ \lambda_{c\to\text{employed}} &= \psi \lambda (1 - u) S. \end{aligned} \tag{1}$$

We omit the arguments of $S(\cdot)$ in what follows. The expression for S reveals the special importance of the quadratic contact technology, since then $S = 1$ and does not depend on u and v. Another important special case is when off- and on-the-job search are equally efficient, $\psi = 1$, since then $S = v^{\xi - 1}$, and does not depend on u, so that the decomposition of labor supply in employed and unemployed job seekers is irrelevant.

Job separation

Matches between workers and jobs are destroyed at an exogenous rate $\delta > 0$.

Wage setting

We focus on two wage-setting schemes. First, we assume that wages are determined by bilaterally efficient bargaining between the worker and the firm where firms cannot

[5] Recall that λ consists of a size-of-the-market part and a search-efficiency part, $\mu(t)$. We assume that the product of both, $\mu(t) L(t)^{\eta + \xi - 1}$, remains constant over time. This implies that $\mu(t) = \lambda \exp[-(\eta + \xi - 1)\rho t]$.

commit on future wage payments (see Diamond, 1982, Mortensen, 1982 and Pissarides, 2000). Second, we assume that firms do not observe the previous employer of the worker (and therefore cannot condition their wages on the value of x in the previous job), but that they can commit on a future wage policy that is contingent on x in the current job. In both frameworks, firms pay "no-quit" premia, but only in the wage-posting model firms can commit ex ante to pay "hiring premia", as in Burdett and Mortensen (1998) and Mortensen (2000). In bargaining models, where wages are continuously renegotiated, hiring premia are not credible because they will be immediately eliminated at the moment the worker starts his job (since from this moment onwards, the worker's outside option is unemployment). Since workers anticipate this, they will not respond to such premia; so firms will thus not offer them in the first place. "No-quit" premia are credible even without commitment because it is in the firm's interest to pay them as soon as the worker has accepted the job. Starting to pay a no-quit premium only when the worker gets an offer is non-credible again, since the firm has no incentive to continue paying the no-quit premium after the worker has declined the offer. Hence, the only way for the firm to gain credibility in paying no-quit premia is to pay these premia right from the start of the job. Section 4.4 discusses the implications of Bertrand competition between the poaching firm and the incumbent firm (as in Postel-Vinay and Robin, 2002 and Moscarini's, 2005 wage mechanisms) for our results.

Distribution of vacancies

We assume that vacancies are uniformly distributed over the circumference of the circle: $v(c) = v$, where $v(c)$ is the number of vacancies per unit of c (that is: the unnormalized density of vacancies at c). Gautier et al. (2005) proves that this is the case for a similar model without on-the-job search. Here, we show that when we allow for on-the-job search, this uniform distribution of vacancies is an equilibrium, but we cannot prove that no other equilibria exist.[6] Since we focus on equilibria where both supply and demand are uniformly distributed, all outcome variables do not depend on either s or c separately, but only on the distance x between them. Hence, we use x as the only argument. For

[6]It is very likely that the uniqueness result carries over to the case with on-the-job search. The intuition is that wages should be high at locations with relatively many vacancies. This is due to both the increase in the outside option of the unemployed workers and the increase in the no-quit premium. The high wages at these locations suggest that the value of a vacancy is relatively low there, which violates the assumption of free entry. Hence, a situation in which some locations have more vacancies than others cannot be an equilibrium.

variables depending only on either s or c (such as $v(c)$), the argument is dropped to simplify notation.

2.1 Job dynamics and asset values

As long as wages are a decreasing function of the mismatch indicator x, the pattern of job dynamics can be solved independently from the wage-setting rule because workers will always accept a job offer with a higher payoff than their current job type. We therefore first solve for the worker flows and the asset values of vacancies, employment and unemployment, conditional on a wage setting rule. The asset values take a simple and easily interpretable form which applies in any equilibrium where wages are a decreasing function of x. Then, we discuss the most common wage-setting mechanisms in the literature. In all of those mechanisms, wages are indeed decreasing in x, as we prove in Proposition 1 below. This allows us to write the wage schedule as a function of one variable: $W(x)$ rather than $W(s,c)$.

Let $G(x)$ be the fraction of workers employed in jobs at smaller distance from their optimal job than x.[7] Let $\overline{x}$ be the largest acceptable distance from their optimal job for unemployed workers. Job offers with a larger x are declined. Hence $G(\overline{x}) = 1$, since all matches have a smaller x than $\overline{x}$. At the golden-growth path, unemployment and employment must grow at the same rate. The inflow into the class of employed workers at distance x or less from their favorite job minus the outflow from this class must therefore equal the population growth rate:

$$2\lambda vSx\left\{u+\psi(1-u)\left[1-G(x)\right]\right\}L(t)-\delta(1-u)G(x)\,L(t)=\rho(1-u)G(x)\,L(t). \quad (2)$$

The first term on the left-hand side is the number of people that finds a job with mismatch indicator less than x, either from unemployment (the first term in parentheses), or by mobility from jobs with a larger mismatch indicator (the second term in parentheses). The number of better jobs is given by $2vx$, since the worker can accept jobs both to the left and to the right of her favorite job type $s=c$ (or, equivalently, $x=0$). The second term in brackets is weighted by the factor ψ, reflecting the efficiency of on-the-job search relative to search while unemployed. The final term on the left-hand side is the outflow

[7] We assume that if a worker is offered a new job with the same payoff as his current job she moves. All results continue to hold if we assume that she moves with an arbitrary small probability; see also Shimer (2006).

of workers from the class of matches with mismatch indicator of x or less: $\delta G(x)$. The right-hand side reflects that at the balanced growth path, employment grows at a rate ρ at all levels including the class of workers with a mismatch indicator smaller than x, $G(x)$. Mobility within this class is irrelevant because the disappearance of the old match and the emergence of the new one cancel. Evaluating (2) at $\overline{x}$ gives

$$\delta(1-u)L(t) + \rho L(t) - u 2\lambda v S \overline{x} L(t) = u \rho L(t).$$

The first term on the left-hand side is unemployment inflow due to job destruction, and the second term on the left-hand side is the size of the new cohort (remember that all workers start unemployed). The third term is the flow of workers who find an acceptable job. The right-hand side is the growth of unemployment that makes u constant over time. Solving for u gives

$$u = \frac{1}{1+\kappa v S \overline{x}}, \qquad (3)$$
$$\text{where } \kappa \equiv \frac{2\lambda}{\rho+\delta}.$$

Note that on the balanced-growth path, u and $G(x)$ are constant. Substitution of (3) for u in the balanced-growth condition (2) yields

$$G(x) = 1 - \frac{\overline{x} - x}{(1+\psi\kappa v S x)\,\overline{x}}. \qquad (4)$$

Hence, the density of the mismatch indicator reads

$$g(x) = \frac{1+\psi\kappa v S \overline{x}}{\overline{x}\,(1+\psi\kappa v S x)^2}. \qquad (5)$$

Let $W(x)$ be the wage for a job with mismatch indicator x and let V^U be the asset value of unemployment. For the two wage mechanisms that we consider, we can write the asset value for a worker holding a job at distance x from his optimal job type, $V^E(x)$ as

$$\rho V^E(x) = W(x) + 2\psi\lambda v S \int_0^x \left[V^E(z) - V^E(x)\right] dz - \delta\left[V^E(x) - V^U\right]. \qquad (6)$$

The disadvantage of writing the asset value this way is that it yields an implicit equation in $V^E(x)$. In Appendix A.2 we show that the asset value can also be written in explicit form:

$$V^E(x) = \frac{W(x)}{\rho+\delta+2\psi\lambda v S x} + \frac{\delta}{\rho+\delta} V^U + 2\psi\lambda v S \int_0^x \frac{W(z)}{(\rho+\delta+2\psi\lambda v S z)^2} dz. \qquad (7)$$

where the first term is the discounted wage income at the current job. The discount factor consists of a time- preference term, ρ, and two terms that take into account the rate at which employed workers leave their jobs (either to unemployment at rate δ, or to a better job at rate $2\psi\lambda v S x$. The second term is the probability of moving to the state of unemployment times the properly discounted value of this state, and the final term is the probability of finding a better job times the discounted expected wage at this better job. The asset value for an unemployed job seeker satisfies the Bellman equation:

$$\rho V^U = B + 2\lambda v S \int_0^{\overline{x}} \left[V^E (x) - V^U \right] dx. \tag{8}$$

For the marginal job, $x = \overline{x}$, the asset value of the job is equal to the asset value of unemployment: $V^E(\overline{x}) = V^U$. Let $\mathrm{E}_G W \equiv \int_0^{\overline{x}} g(x) W(x) dx$ be the expected wage of a filled job. Evaluating (7) at $\overline{x}$ and using the definition of g in (5) gives

$$\rho V^E(\overline{x}) = \rho V^U = \frac{W(\overline{x}) + \psi\kappa v S \overline{x} \mathrm{E}_G W}{1 + \psi\kappa v S \overline{x}} = \frac{u W(\overline{x}) + \psi (1-u) \mathrm{E}_G W}{u + \psi (1-u)}. \tag{9}$$

The asset value in the marginal job is a weighted average of the wage in the current job $W(\overline{x})$ (the reservation wage) and the expected wage in the next job, $\mathrm{E}_G W$. The latter reflects the option value of on-the-job search. Using the fact that the right-hand side of (6) and (9) are equal allows us to obtain an expression for $\int_0^{\overline{x}} \left[V^E(x) - V^U \right] dx$. Substitution of this expression into (8) allows us to write ρV^U as a function of $W(\overline{x})$, which can be substituted out by solving (9) for $W(\overline{x})$. This gives

$$\rho V^U = \frac{B + \kappa v S \overline{x} \mathrm{E}_G W}{1 + \kappa v S \overline{x}} = uB + (1-u) \mathrm{E}_G W, \tag{10}$$

where the final step uses (3). Equation (10) has a very appealing interpretation: the asset value of unemployment is equal to a weighted average of the value of leisure B and the expected wage in all jobs, the weights being equal to the rates of unemployment and employment, respectively. The simplicity of (10) is due to the golden-growth assumption, which implies that the weight of the value of leisure in the asset value of unemployment is equal to the unemployment rate. In a zero-population growth economy, the weight of the value of leisure would exceed the unemployment rate, since the expected payoff of future employment should be discounted and hence carries less weight. In a growing economy, new entrants enter the economy as unemployed, so that the unemployment rate is higher. This effect exactly offsets the effect of discounting future payoffs.

Since the right-hand sides of equations (9) and (10) are equal, we obtain (after some rewriting) the following:

$$W(\overline{x}) = B\left[u + \psi\left(1-u\right)\right] + \left(1-\psi\right)\left(1-u\right)\mathrm{E}_G W. \tag{11}$$

The reservation wage is a weighted average of the value of leisure and the expected wage in employment. The latter term reflects the additional option value of search when unemployed. At the marginal job, $x = \overline{x}$, the surplus $Y(\overline{x}) - W(\overline{x})$ must be zero because if $Y(\overline{x}) - W(\overline{x}) > 0$, there would be a value of $\tilde{x} > \overline{x}$ such that $Y(\tilde{x}) > W(\overline{x})$. Both firms and workers would then accept a wage offer $W(\tilde{x}) = \frac{1}{2}\left[Y(\tilde{x}) + W(\overline{x})\right] > W(\overline{x})$ for this job, but that contradicts $\overline{x}$ being the marginal job. Obviously, $Y(\overline{x}) - W(\overline{x})$ cannot be negative, because then the firm would be better off by not making a wage offer at all. Hence,

$$W(\overline{x}) = Y(\overline{x}) = 1 - \frac{1}{2}\gamma\overline{x}^2. \tag{12}$$

By the free entry condition for firms, the option value of a vacancy of type c must be equal to K. Hence, by defining $\mathrm{E}_G Y \equiv \int_0^{\overline{x}} g(x)\, Y(x)\, dx$, we get

$$\begin{aligned} K &= 2\lambda S \int_0^{\overline{x}} \left\{u + \psi\left(1-u\right)\left[1 - G(x)\right]\right\} \frac{Y(x) - W(x)}{\rho + \delta + 2\psi\lambda v S x} dx \\ &= \frac{1-u}{v}\left(\mathrm{E}_G Y - \mathrm{E}_G W\right). \end{aligned} \tag{13}$$

Starting off with the first equality, the first factor in the integrand is the effective fraction of individuals $u + \psi(1-u)[1-G(x)]$ willing to accept a type x match. It equals the number of unemployed, u plus the number of workers employed in jobs with a greater mismatch indicator than x, $(1-u)[1-G(x)]$; the latter number is scaled down with ψ to account for a lower effectiveness of on-the-job search. The second factor is the value of a filled vacancy. Just as in the wage equation, we discount current revenue $Y(x) - W(x)$ by the discount rate ρ plus the separation rate δ plus the quit rate $2\psi\lambda v x$. The second line follows from substitution of the relations for employment and unemployment. Multiplying (13) by v gives a simple interpretation of this zero-profit condition: at any point in time, the cost of vacancies, vK, is equal to the employment rate $(1-u)$ times the expected profit in a filled vacancy, $(\mathrm{E}_G Y - \mathrm{E}_G W)$.

Equations (3), (7), (10), (11) and (13) and the functional equation (5) determine u, v, ρV^U, $\overline{x}$ and $G(x)$. Only the wage function $W(x)$ (which implies $\mathrm{E}_G W$) remains to be determined.

2.2 Wage setting without commitment

For the derivation of the wage-setting relation, it is useful to define the asset value of employment as a function of wages, W, instead of the mismatch indicator x. Let $\hat{V}^E(W)$ denote this asset value and let $\widehat{F}(W)$ be the fraction of acceptable vacancies that pay a wage equal to or higher than W. This asset value satisfies

$$\rho\hat{V}^E(W) = W + \delta\left[V^U - \hat{V}^E(W)\right] - 2\psi\lambda v\overline{x}S\int_W^\infty \left[\hat{V}^E(Z) - \hat{V}^E(W)\right] d\widehat{F}(Z).$$

The minus sign in front of the last term comes from the fact that $\widehat{F}(Z)$ is a survival function. Throughout the paper, subscripts of functions denote the relevant (partial) derivative. A sufficient condition for equilibrium is that the wage $W(x)$ paid to a worker holding a job with mismatch indicator x maximizes the following product:[8]

$$W(x) = \arg\max_W \left[\hat{V}^E(W) - V^U\right]^\beta \left[\frac{Y(x) - W}{\rho + \delta + 2\psi\lambda v\overline{x}S\widehat{F}(W)}\right]^{1-\beta}.$$

Our solution is consistent with Shimer's alternating offer game, where we interpret the $\beta's$ either as different discount factors or as probabilities to make an offer. In his setting, an equilibrium wage is an extremum of this product. The first factor in square brackets is the increase in wealth for the worker relative to the status of unemployment, and the second factor is the increase in wealth for the firm. The latter is the current income stream $Y(x) - W$ divided by a modified discount rate, which consists of the discount rate ρ, the separation rate δ, and the quit rate of the worker to better-paid jobs. The latter equals the probability that a worker finds an acceptable job $2\psi\lambda v\overline{x}S$ times the probability that this job pays more that the current wage W, which equals $\widehat{F}(W)$. By paying higher wages, firms and workers reduce the probability that the worker quits to a better-paying job. This increases the surplus product in the present job. Hence, firms pay a no-quit premium, which raises the wage above the simple sharing rule that applies in models without on-the-job search; see also Shimer (2006).

The function $\widehat{F}(W)$ is the survival function of the wage-offer distribution. This distribution has no mass points for the usual reasons; see Burdett and Mortensen (1998). If $\widehat{F}(W)$ would have a mass point at W^*, then a profitable deviation exists. The surplus product would jump upward by a slight increase in W above W^*, since all vacancies at the mass point would no longer be able to poach a worker from the job paying this slightly

[8] At this stage, we cannot rule out that the set of feasible pay-offs is non-convex.

higher wage. This contradicts W^* being a maximum of the surplus product. Since the surplus product is continuous in W, and since both $Y(x) - W(x)$ and $\hat{V}^E[W(x)] - V^U$ are positive (otherwise, either the firm or the worker would not be willing to match), $W(x)$ satisfies the following first-order condition:

$$\beta[Y(x) - W(x)] = (1-\beta)\left[\hat{V}^E[W(x)] - V^U\right] \times \left[\rho + \delta + 2\psi\lambda v\overline{x}S\widehat{F}[W(x)] + 2\psi\lambda v\overline{x}S\widehat{F}_W[W(x)][Y(x) - W(x)]\right],$$

where we use $\hat{V}^E_W(W) = \left[\rho + \delta + 2\psi\lambda v\overline{x}S\widehat{F}(W)\right]^{-1}$. This condition states that the gain from a marginal wage increase for the worker is equal to the cost of that increase for the firm, where both are weighted by their respective bargaining power β and $1-\beta$. The following proposition allows us to define the number of vacancies and the asset value of a job as functions of x instead of W:

Proposition 1 *The function $W(x)$ is decreasing on $(0, \underline{x})$.*

Proof. See appendix A.1.

Therefore, we can write

$$V^E(x) \equiv \hat{V}^E[W(x)],$$
$$\frac{x}{\overline{x}} = \widehat{F}[W(x)].$$

In the second line, we use the uniformity of the distribution of x among vacancies. For a given worker type, $\widehat{F}[W(x)]$ is equal to the number of jobs located at smaller distance than x as a fraction of all acceptable jobs. By the chain rule for differentiation, we obtain:

$$V^E_x(x) = \hat{V}^E_W[W(x)]\,W_x(x),$$
$$\frac{1}{\overline{x}} = \widehat{f}[W(x)]\,W_x(x).$$

Substituting this expression in the first-order condition and rearranging terms yields

$$W_x(x) = \frac{(1-\beta)\psi\kappa vS[Y(x) - W(x)]R}{\beta[Y(x) - W(x)] - (1-\beta)(1+\psi\kappa vSx)R}, \tag{14}$$

where $R \equiv \left[V^E(x) - V^U\right](\rho+\delta)$. This is a differential equation in x. Its solution requires an initial condition, which is given by (12). When $\beta = 0$, we obtain

$$W_x(x) = -\frac{\psi\kappa vS[Y(x) - W(x)]}{1+\psi\kappa vSx} \tag{15}$$

Definition 2 *The equilibrium without commitment consists of the set* $\{u,\ v,\ \overline{x},\ W(\overline{x}),\ W(x),\ G(x)\}$ *satisfying the equations (3), (11), (12), and (13), and functional equations (5) and (14).*

2.3 Wage setting with commitment

When firms can commit to future wage payments, it is also straightforward to show that $W(x)$ is decreasing in x. Since the proof is almost identical to the one in appendix A.1, we omit it here. The main difference between the wage mechanism under commitment and the one in the previous section is that commitment allows firms to maximize the expected value of a vacancy. We allow firms to post a wage conditional on the distance between worker and job type, x. Note that this commitment assumption is stronger than in the homogeneous case where firms only have to commit to a wage. Here, firms have to commit to a wage policy contingent on x. This requires that x is verifiable at the moment the contract is signed. For the same reasons as in the previous section, one can show that $W'(x) < 0$. Since the proof is similar to the proof of proposition 1 in appendix A.1, we omit it here. By offering a higher wage, firms can increase their expected labor supply. Let $\widehat{G}[W(x)] \equiv G(x)$ be the fraction of employed workers who currently earn less than $W(x)$. Then

$$W(x) = \arg\max_{W} \left(\left\{ u + \psi (1-u) \left[1 - \widehat{G}(W) \right] \right\} \times \frac{Y(x) - W}{\rho + \delta + 2\psi\lambda v \overline{x} S \widehat{F}(W)} \right). \quad (16)$$

$u + \psi (1-u) \left[1 - \widehat{G}(W) \right]$ is the probability that the job seeker accepts the wage offer. Unemployed job seekers will always accept the job, and employed job seekers accept the job when their current job pays less than W, with the probability that this occurs increasing in W. The current payoff of hiring a worker at distance x equals $Y(x) - W$, which is decreasing in W. Finally, the quit rate $\psi\lambda v S \widehat{F}(W)$ is decreasing in W, which means that the effect of W on the value of the vacancy via the quit rate is positive. The optimal wage offer $W(x)$ balances these negative and positive effects of W on the value of the vacancy. Using equation (4) and rearranging terms, we can write the first-order condition as

$$W_x(x) = -2 \frac{\psi\kappa v S}{1 + \psi\kappa v S x} \left[Y(x) - W(x) \right]. \quad (17)$$

See appendix A.6 for a derivation.[9] If we compare (17) with (15) for $\beta = 0$, we see that the only difference is the factor 2 in (17), which is absent in (15). This reflects the fact that under a wage-setting regime with commitment, firms not only pay a no-quit premium but also a hiring premium. This hiring premium is exactly equal to the no-quit premium, because the value of a given match is the same, irrespective of whether the job is occupied by a new or an existing worker.

Definition 3 *The equilibrium with commitment consists of the set* $\{u,\ v,\ \overline{x},\ W(\overline{x}),\ W(x),\ G(x)\}$ *satisfying equations (3), (11), (12), (13) and functional equations (5) and (17).*

3 Characterization of the equilibrium

This section characterizes the equilibrium for various cases. Section 3.1 discusses the on-the-job search equilibrium where firms cannot commit to future wage payments. We consider the simplest case where firms have all the bargaining power (monopsony without commitment), $0 < \psi \leq 1$, $\beta = 0$. We pay special attention to the $\psi = 1$ case, where on- and off-the-job search are equally efficient. This case is relatively simple because the reservation wage of an unemployed job seeker reduces to $W(\overline{x}) = B$ because accepting a job does not imply a reduction in the option value of search. The general case $0 < \psi < 1, 0 < \beta < 1$ is hard to analyze analytically because an explicit solution for the wage function $W(x)$ is not available. Section 3.2 considers a wage mechanism where firms can ex ante commit to future wage payments.

3.1 Wage setting without commitment

The solution of the differential equation (15) is (see Appendix A.3 for a derivation)

$$W(x) = 1 + \gamma \frac{x - \overline{x}}{\psi \kappa S v} + \frac{1}{2}\gamma x (x - 2\overline{x}) - \gamma \frac{1 + \psi \kappa S v x}{(\psi \kappa S v)^2} \log \left(\frac{1 + \psi \kappa S v x}{1 + \psi \kappa S v \overline{x}} \right). \tag{18}$$

The wage equation (18) is dramatically different from the bargained wage in a stochastic job-matching model without on-the-job search; see Pissarides (2000). In the latter case, a simple linear sharing rule applies, where wages are a weighted average of the outside option

[9] Bontemps, Robin and Van den Berg (2000) show in a similar setting that no mixed strategy equilibria exist.

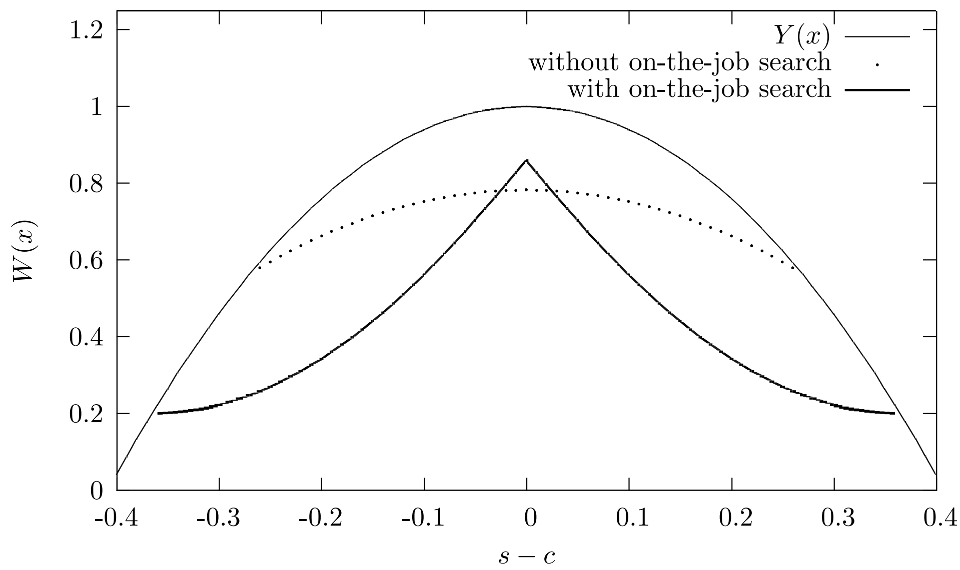

Figure 1: The wages paid by firms as a function of the levels of x. The parameters used are $\gamma = 12, B = 0.2, K = 0.1, \kappa = 10$ and $\psi = 1, \beta = 0$ for the case with on-the-job search, while $\psi = 0, \beta = 0.5$ for the case without on-the-job search.

of the firm and the reservation wage of the worker. Then, the flatness of productivity in the optimal assignment implies the flatness of the wage function at that point: $W_x(0) = Y_x(0) = 0$. This characteristic does not carry over to the equilibrium with on-the-job search. The situation is sketched in Figure 1. We draw $W(x)$ both for $\psi = 0$ (no on-the-job search, $\beta = 0.5$; see the working paper version for a derivation) and $\psi = 1$ ($\beta = 0$). The locus of value added $Y(x)$ does not depend on ψ. On-the-job search lowers the reservation wage $W(\overline{x})$, since employed workers retain the option value of search so that accepting a job is less costly and matching sets become larger. Firms pay no-quit premia to prevent their workers to be poached by other firms. The premium paid by one firm induces other firms to pay even higher premia in equilibrium. As a consequence, the wage locus is non-differentiable at the optimal assignment, $x = 0$, leading to a peak in the locus. Since all firms pay no-quit premia, there is no net effect on actual quit behavior. So even though firms compete for workers by paying no-quit premia, the actual mobility pattern is unaffected by these premia: workers keep on moving towards the optimal assignment. Any vacancy type with a lower x than the present job will be accepted.

The consequence of this argument is that the instantaneous profit margin for an employed worker, $Y(x) - W(x)$, does not reach its maximum at the optimal assignment,

$x = 0$: while $Y_x(0) = 0$, $W_x^+(0) < 0$ (and the reverse for the left derivative). Hence, an ε deviation leads to a rise in the surplus. The effect of on-the-job search on $W(x)$ is indeterminate. Since reservation wages are lower with on-the-job search, wages for similar jobs can be either lower or higher: the no-quit premium pushes wages up, the lower reservation wage pulls them down.

Proposition 4 *Define*

$$K^* \equiv \frac{\sqrt{\gamma}K}{\kappa(1-B)^{3/2}}.$$

For (i) $\xi < 1$ and for (ii) $\xi = 1$ and $K^* < \frac{2}{3}\sqrt{2}$ there exists an equilibrium with a positive supply of vacancies. For $\psi = 1$ the equilibrium is unique.

Proof: See Appendix A.4.■

Though we have not been able to find a general proof of uniqueness, the propositions for the upper- and lower bounds of ψ and plots for intermediate values suggest that uniqueness holds for any combination of parameters.[10] Shimer (2006) shows that multiple equilibria can arise if workers always remain at their current job if they receive an equal offer and/or if workers can reject offers and make counteroffers.

The comparative statics results are given below in Table 1; see Appendix A.5 for the derivation.

	u	v
γ	$+$	$\cup(-)$
κ	$-$	$\cap(+)$
B	$+$	$-$
K	$+$	$-$
β	$+$	$-$
ξ	$-$	$-$
η	$-$	$-$

Table 1: Comparative statics of the model for $\beta > 0$ and $\psi = 0$. A "+" sign indicates that the first-order derivative is positive. The $\cap$-sign implies that the first-order derivative is positive for small levels of u and negative for larger values, and the $\cup$-sign implies the opposite situation. We give the expected sign for realistic values of the unemployment rate in parentheses.

[10]For $\psi = 0$, uniqueness can be proved.

We find that the sign of the impact of v on γ and κ depends on the level of the unemployment rate (the expected signs appear in parentheses in the table). The unemployment rate should be over 27% in order to falsify these expected signs of v and κ and v and γ, which is an irrelevant range. Hence, we take these effects as a benchmark. If γ increases, it is more important to have a good match, and workers and firms will become more choosy. This makes it less attractive to open vacancies, implying that unemployment increases. The other comparative statics effects are as expected. Details are contained in Appendix A.5.

3.2 Wage setting with commitment

Like the case without commitment, the differential equation for wage setting under wage posting, equation (17) can be solved analytically (see Appendix A.6 for a derivation):

$$\begin{aligned} W(x) = 1 - \frac{3}{2}\gamma x^2 + \frac{\gamma\overline{x}}{\psi\kappa S v}\frac{(1+\psi\kappa S v x)^2}{1+\psi\kappa S v\overline{x}} - \frac{\gamma x}{\psi\kappa S v} \\ - \gamma\left(\frac{1+\psi\kappa S v x}{\psi\kappa S v}\right)^2 \log\left(\frac{1+\psi\kappa S v\overline{x}}{1+\psi\kappa S v x}\right). \end{aligned} \tag{19}$$

Figure 2 plots the wage function with and without commitment, both for $\psi = 1$. Because of the additional hiring premia, wages are strictly higher in the commitment case. The possibility of committing to a wage increases the profits for an individual firm. However, if all firms have this option, equilibrium profits will be lower than when firms cannot commit. Hence, for a given stock of vacancies, commitment makes firms worse off and workers better off, because it increases competition.

Appendix A.7 proves that Proposition 4 also holds for the case of wage setting with commitment. This implies that the conditions for a positive supply of vacancies do not depend on the wage-setting regime. Again, we are not able to prove uniqueness of the equilibrium for the general case, but plots for the other values of ψ suggest that uniqueness applies generally. The comparative statics for the case of $\psi = 1$ are qualitatively the same as in Table 1 in Section 3.1. Details appear in Appendix A.8.

4 Efficiency

In a world with search frictions, output is lower than in a Walrasian world. There are two reasons why search frictions reduce output. First, the constraints imposed by the

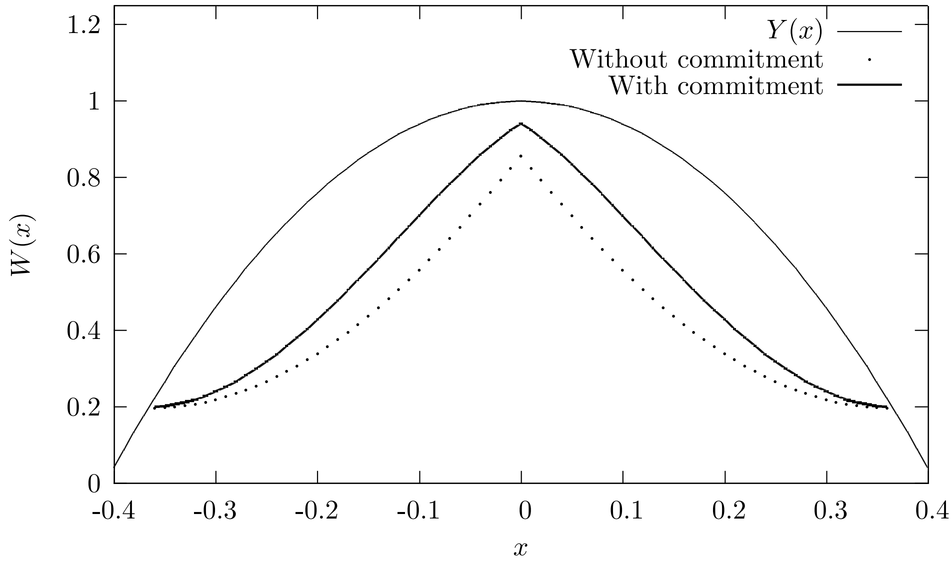

Figure 2: Wages with and without commitment. Parameter values are the same as in Figure 1.

search technology cause unemployment, mismatch and costly vacancy creation. Second, output is lost due to inefficient decentralized decision making, given the constraints of the search technology. An equilibrium is constrained efficient when this second source of inefficiency is equal to zero. This section presents the efficiency properties of the various models and suggest institutional remedies to reduce or even eliminate the inefficiencies such as changing workers' bargaining power β or introducing unemployment insurance, so that agents' decisions are better aligned.

Define net output Ω as:

$$\Omega \equiv (1-u)\,\mathrm{E}_G Y + uB - vK. \tag{20}$$

The first term is actual output of the employed workers, the second term captures the value of leisure of the unemployed job seekers, and the third term is the cost of keeping vacancies open. The subsequent lemma relates this expression to the asset value of unemployment in the decentralized equilibrium.

Proposition 5

$$\Omega = \rho V^U. \tag{21}$$

Proof: Substitution of equations (10) and (13) in equation (20) proves the Proposition.■

Proposition 5 simplifies our welfare analysis; it suggests that we can just maximize Ω in the steady state here.[11,12]

Substitution of the definition of $\mathrm{E}_G(Y)$ and (3) in (20) yields an expression for Ω as a function of the acceptance rule $\overline{x}$ and unemployment u. This expression depends only on technical constraints, not on decision rules.

Given the constraints imposed on Ω by the search technology, the market outcome depends on three types of decisions: (1) the number of vacancies v opened by firms, (2) the job acceptance rule $\overline{x}$ applied by unemployed job seekers, and (3) the job acceptance rule applied by employed workers. The latter decision rule is simple in this model. Employed workers accept any job offer that pays a higher wage than their current job. Since all wage-setting rules considered in this paper imply that wages are a decreasing function of x, $W_x(x) < 0$, workers by implication, accept any job that is at a shorter distance from the optimal assignment than their current job. This decision rule is clearly efficient, since there is no option value lost by switching to a more productive job, for neither the worker, nor the firm. This leaves the social planner with two remaining decision variables, $\overline{x}$ and v. Since equation (3) provides the steady-state relation between v, on the one hand, and $\overline{x}$ and u, on the other, we can also use the latter two as the relevant decision variables. The social planner's first-best optimum maximizes Ω with respect to $\overline{x}$ and u.

The general case of $\psi \in [0, 1]$ is hard to analyze. Hence, we focus on the special case where on- and off-the-job search are equally efficient. It implies that $W(\overline{x}) = Y(\overline{x}) = B$ in the decentralized equilibrium. Any job offer that pays more than the flow value of unemployment will be accepted. This rule is clearly efficient, since job seekers do not give up any option value from extended job search by accepting a job because on- and off-the-job search are equally efficient. Again, we can focus on the optimality of unemployment, since $\overline{x} = \overline{x}^*$, anyway. We analyze efficiency conditions for the case where on- and off-the-job search are equally efficient and $\psi = 1$, and then discuss institutional settings that improve efficiency.

[11]The fact that the Planner's problem is equivalent to maximizing Ω in the steady state does not hold in general. It holds here because of the golden-growth assumption. Future output is discounted but also receives a higher wage because the population grows. Those effects cancel. A similar result holds if $\rho/\delta \to 0$, as in Burdett and Mortensen (1998).

[12]Pissarides (2000) uses a model without on-the-job search to show that maximizing the net discounted value of all future output is equal to maximizing the asset value of unemployment.

4.1 Wage setting without commitment

We start again with the simplest case, where $\beta = 0$. In order to eliminate the well-known congestion externalities, we set $\psi = \xi = 1$.

Proposition 6 *For the case $\psi = \xi = 1$, v is above its socially efficient level.*

Proof: see Appendix A.9.■

The intuition behind the inefficiency is that when a firm opens a vacancy it does not adequately internalize the expected output loss that it imposes on the firm from which it poaches a worker. Although the job switch is efficient (as we discussed before), the social contribution of a vacancy is too low to justify its cost. This externality is different from the poaching externality that is driven by investments in human capital (i.e. Moen and Rosen, 2004). In that case, poaching reduces investments in human capital because parts of the returns to these investments go to future employers. Although our model has no investments in human capital, the equilibrium is still inefficient. Our finding is related to the "business-stealing" externality in for example, Salop (1979) and Dixit and Stiglitz (1977). In these models, economics of scale are not optimally exploited. In our setting, the expected private benefits of opening a vacancy are higher than the social benefits. For $\xi < 1$, the overinvestment in vacancies gets worse because now firms also do not take into account the negative congestion externality that they impose on other firms. From an analytical point of view, the case $\xi = 1$ is instructive, as it allows us to analyze the business-stealing effect in isolation by setting the congestion externalities equal to zero.

Figure 3 plots v against κ for the market and the planner. Both figures are non-monotonic in κ. The turning point of the social planner lies to the right of the turning point of the market. The reason why the excess number of vacancies is hump shaped can be understood by considering the upper- and lower bounds of κ. As $\kappa \to \infty$, $v \to 0$ and $u \to 0$, both for the market and the planner the equilibrium is trivially constrained efficient because the frictions are irrelevant. As $\kappa \to 0$, $v \to 0$ and $u \to 1$, on-the-job search becomes only marginally important compared to search by unemployed job seekers, simply because all workers are unemployed. In that case, business stealing can obviously not be an important phenomenon. For the intermediate cases, vacancy supply is excessive under monopsony, due to the business-stealing effect.

For the case $\psi = 1$ there exists a value of $\beta \in (0, 1)$ for which the decentralized unemployment rate equals the social planner's optimum. We already showed that unem-

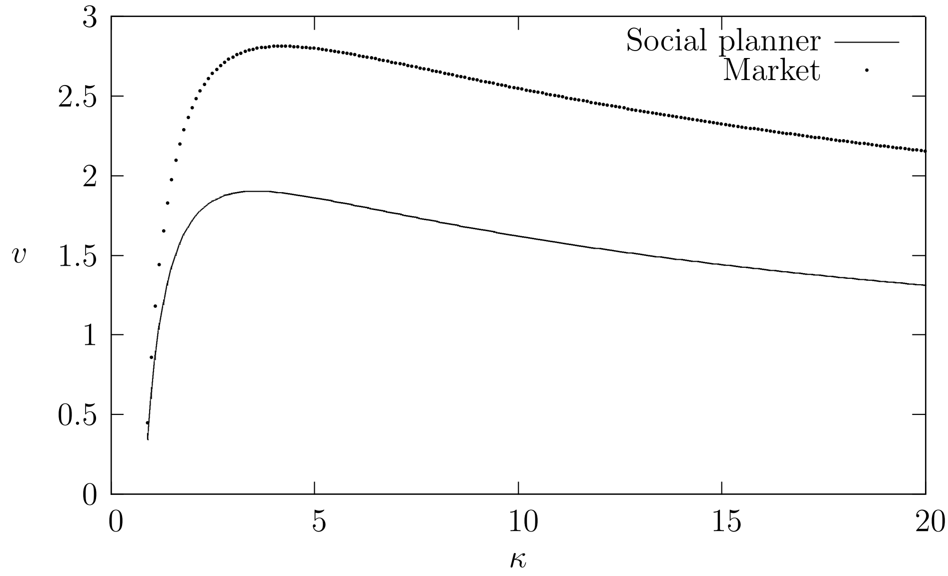

Figure 3: Planner's and decentralized market vacancy rate as a function of κ for $\psi = \xi = 1$.

ployment is too low for $\beta = 0$. For $\beta = 1$, workers receive the full match surplus. This drives the number of vacancies to zero and unemployment to one, which is obviously too high. Now consider the zero-profit condition for $\psi = 1$:

$$vK = (1 - u)\left(\mathrm{E}_G Y - \mathrm{E}_G W\right).$$

Since $\overline{x}$ does not depend on β for $\psi = 1$, and since $W(x)$ is determined by (14), which is continuous in β, vacancy supply is continuous in β. By the continuity of vacancy supply in β, there must be an intermediate value for β for which the unemployment rate is equal to the social planner's optimum.

The optimal value of β can be calculated numerically in the following way. First, we calculate the desired number of vacancies in the planner's equilibrium. Then, we determine numerically the level of β that yields this desired number of vacancies, using (13) and (14). We find that the level of β needed to offset the "business-stealing" externality is quite large (for reasonable parameter values, β must be close to 0.5). The lower $\frac{1-B}{K}$, the higher the required value of β, since vacancy creation costs are then high and most workers are already close to the optimal assignment, so the "business-stealing" externality is large.

Finally, recall that we switched off the congestion externalities by considering a quadratic contact technology; if we would switch them on, there would no longer be a β that gives first best. Marimon and Zilibotti (1999) show that with a Cobb Douglas contact technology, the Hosios condition ($\beta = \eta$) restores efficiency. The difference between the business-stealing externality and the congestion externality is that under a congestion externality, when a firm hires an unemployed worker, other firms are hurt (since they can no longer hire this worker), but the other unemployed workers gain (because they experience less congestion). However, the business-stealing externality hurts other firms but does not make the other unemployed workers better off.

The above analysis has shown that raising β above zero raises efficiency by reducing the incentives to create vacancies. However, if the social planner has no instrument to change the bargaining power of the workers, then the introduction of unemployment insurance can be an alternative. From the outset we can see that this instrument can never implement first best because it distorts the job-acceptance decision because of a moral hazard problem: workers reject all jobs that pay less than the value of leisure plus the unemployment benefit, while the efficient rule would be to reject only jobs that pay less than the value of leisure. Since we consider the decentralized equilibrium, equation (21) applies. For the sake of simplicity, we concentrate on the case in which the value of leisure is equal to zero, so that we can interpret B as an unemployment benefit. In that case, a term uB should be subtracted from Ω, so that the planner maximizes $\Omega - uB$ where B can be thought of as being financed by a lump-sum tax over the entire labor force.

Proposition 7 *The optimal level of UI benefits for the case* $\psi = \xi = 1, \beta = 0$ *is positive,* $B > 0$.

Proof: See Appendix A.10.■

This is hardly a new conclusion. Previously, Burdett (1979), Diamond (1981), Marimon and Zilibotti (1999) and Teulings and Gautier (2004) discussed models with risk-neutral agents where UI benefits can serve as a search subsidy that prevents workers from entering bad matches. This paper provides a different argument; namely, that unemployment insurance reduces the business-stealing externality that leads to excess vacancy supply by raising wages. For the case in which $\xi < 1$ we can also prove that the optimal benefit level is positive.

4.2 Wage setting with commitment

Proposition 8 *The wage-setting mechanism with commitment is constrained efficient when $\psi = \xi = 1$.*

Proof: This follows from the fact that (32) in Appendix A.7 is identical to the social planner's optimal unemployment rate (34) in Appendix A.9.

To the best of our knowledge, the fact that the wage setting with commitment equilibrium is constrained efficient when on- and off-the-job search are equally efficient, is a new result. The fact that workers behave precisely as they should is not surprising because, as in the previous section, all congestion externalities are switched off. The crucial issues are therefore as follows. Why are the firm's expected profits on a vacancy equal to the social value of a vacancy? Why does wage setting with commitment give the socially desired incentives, and why does wage setting without commitment fail to do so? Below we provide some intuition.

Suppose that all firms pay their workers the expected social shadow price $W^*(x)$, conditional on the value of x in that job.[13] By definition, these social shadow prices implement an efficient equilibrium. Then, in the absence of externalities a deviant firm that has the same strategy space as the non-deviant firms cannot realize higher expected profits on its vacancy. The business-stealing externality is absent under commitment because a firm can post a higher wage if their job candidates are on average employed at good jobs than when they are employed at bad jobs. In equilibrium, the premium that a firm is willing to offer to get an extra hire at distance x must be equal to the premium that the firm is willing to pay to prevent a quit of an $x-$type match. The equilibrium wage schedule that we derived for the no-commitment case cannot be equal to $W^*(x)$. This follows from the fact that a firm that is able to commit will be able to make more profits by offering higher wages.[14] With commitment, firms offer the expected

[13]The shadow value of a worker equals her expected discounted future production conditional on her current state. The social contribution of an x match with a worker who came from a z match is thus the difference between the two shadow values. The shadow wage is the wage that the firm that offers the x match must pay in order to receive the social contribution of this match. The expected shadow wage, $W^*(x)$ is the wage schedule that makes the firms' payoffs equivalent to this at the moment the vacancy is opened.

[14]Note that although a single deviant firm that is able to commit would make a higher profit than all other firms, the profits of all firms would go down if all firms were able to commit. The reason is that the

shadow wage, $W^*(x)$ because that maximizes the expected payoff of a vacancy. Since firms open vacancies until expected profits are equal to zero, the social value of adding another vacancy is zero; that is, the economy is in its constrained efficient equilibrium. The central reason why wage setting without commitment is not socially efficient is that the lack of ability to commit constrains the firms' strategy space and keeps them from maximizing the expected value of a vacancy.

4.3 Other wage-setting mechanisms

Postel-Vinay and Robin (2002) (abbreviated here as PR) assume that if an employed worker finds a new job, then there must be Bertrand competition for her services. So in this scheme firms do pay positive hiring premia and at least partly internalize the business-stealing externality. However, as shown in appendix A.11, the resulting equilibrium is still inefficient because the option to match outside offers eliminates the need to pay a no-quit premium. For any value of K^*, vacancy creation in the PR model is most excessive, implying that firms receive the largest share of the surplus. The surplus that the firm extracts from a match in the PR model includes not only current output but also the value of future job search opportunities. This allows them to pay below the productivity level of the firms from which they are poaching workers. Appendix A.11 also shows that a wage scheme is socially optimal when (1) the poaching firm pays the worker her productivity at her previous job and (2) at each offer above a worker's current wage but below her productivity, the firm pays the productivity the worker would realize at the poaching firm. Hence, the wages paid in the PR wage scheme are too low. Basically, rather than paying no-quit premia, the firm is able to appropriate all of the future rents of Bertrand competition. We conclude from this that a requirement for efficiency is that firms pay both hiring and no-quit premia. In bargaining models, the inefficiency arises because firms fail to pay hiring premia whereas in PR, firms fail to pay no-quit premia and pay too little hiring premia.

Dey and Flinn (2005) and Cahuc, Postel-Vinay and Robin (2006) no longer assume that the firm extracts all surplus from a match. Instead, they assume that workers receive a share β of the surplus of the match. Obviously, as in the general version of our model, there will exist a value of β that restores efficiency but, just as with the Hosios condition,

other firms pay their workers a wage $W(x)$ below the expected shadow price $W^*(x)$. Hence, since profits are too high when firms are unable to commit on future wage payments, firms open too many vacancies.

this happens only by coincidence because there is no relation between the value of β and the magnitude of the business-stealing externality and possible congestion externalities.

Moscarini (2005) assumes that if a worker finds a new firm, the wage is determined by a poaching auction where firms are allowed to bid for a wage in combination with a lump-sum hiring premium. He shows that it is a subgame-perfect equilibrium strategy for a firm to bid the worker's bargaining value in its own match and to offer a zero lump-sum payment.[15] Compared to Bertrand competition, the worker receives a higher wage at his first job but lower expected wages in the future.[16] With this wage mechanism there is also excess vacancy creation.

4.4 Wrapping up

Without on-the-job-search, the quadratic contact technology is never efficient in a random-search context because it is impossible to give both workers and firms their contribution to the matching process. Under a CRS contact technology, efficiency is obtained in a bargaining model only for the very special case that the Hosios condition is fulfilled. With on-the-job search, the interaction between the contact technology and the wage mechanism creates interesting new efficiency issues. Under a CRS contact technology, the well-known congestion effects keep the market equilibrium from being efficient, except for very special cases. By assuming a quadratic contact technology, we can switch off the congestion externalities and focus on the new externalities. If firms cannot ex ante commit to a wage schedule, then there will be a business-stealing externality. Under Bertrand competition between the poaching and incumbent firm, the business-stealing externality is partly internalized but in that wage mechanism, the main source of inefficiency is that firms do not pay a no-quit premium. Only the wage-posting-with-commitment equilibrium is efficient from a social welfare point of view because firms pay exactly the right amount of hiring- and no-quit premia.

5 Final remarks

This paper characterizes the equilibrium of a model with a continuum of job and worker types, search frictions, transferable utility and free entry, allowing for on-the-job search.

[15]It is the unique equilibrium strategy if each bid has a cost, ε.

[16]Moscarini shows that under this wage mechanism, the linear sharing rule remains even under Nash bargaining.

On-the-job search has implications for (i) wage bargaining, (ii) entry of vacancies, and (iii) the value of unemployment relative to employment. The existence of on-the-job search explains the stylized fact that unemployed workers move quickly to employment but at the same time are willing to accept relatively modest wage offers. We derived the efficiency implications of on-the-job search for various wage-setting mechanisms, and showed that the number of vacancies is higher and the unemployment rate is lower than in the social optimum if firms cannot commit to future wage payments. This is due to a business-stealing externality: in a market with on-the-job search, an individual firm does not fully internalize that opening a vacancy reduces expected job durations at other matches. For reasonable parameter values this externality turns out to be substantial. Most of our analysis is done for the case where we switched off the congestion externality by using a quadratic job-search technology. Allowing for a congestion externality in combination with monopsony power would make things worse because it would create a second source for excessive vacancy supply.

In a world with two-sided heterogeneity, wage policies with commitment require the match quality to be verifiable when the contract is signed, which is a stronger assumption than the assumption that posted wages are verifiable (as in the Burdett Mortensen case with homogeneous workers).

Positive unemployment benefits can improve efficiency. Interestingly, under wage posting, the business-stealing externality is absent because firms pay higher hiring premia if it is likely that their candidate is currently employed at a productive job. We assumed that search intensity is exogenous and that there are no switching costs. Allowing for endogenous search intensity on the worker's side creates additional sources of inefficiency that have already been studied in the literature. Switching costs also introduce an additional friction. Workers will move only if the expected wage increase also compensates for the switching costs.

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Appendix

A Derivations and proofs

A.1 Proof of proposition 1

We show that $W(x)$ is non-increasing on $(0, \underline{x})$. Consider two values of x, x_0 and x_1 with $x_1 > x_0$. Let $Y_i \equiv Y(x_i)$, so that $Y_0 > Y_1$. In the no-commitment case, the wage W_i

maximizes

$$W_i = \arg\max_W P(W)[Y_i - W],$$
$$P(W) \equiv \frac{1}{1+\psi\kappa S\widehat{F}(W)}.$$

Since $\widehat{F}(w)$ is decreasing in W, $P(W)$ is increasing in W. Let $P_i \equiv P(W_i)$. Since W_0 maximizes $P(W)[Y_i(x) - W]$, we have

$$P_0(Y_0 - W_0) > P_1(Y_0 - W_1). \tag{22}$$

Suppose that, contrary to the proposition, $W_0 < W_1$. Since $P(W)$ is increasing, this implies $P_0 < P_1$ and

$$P_0(Y_1 - W_0) > P_0(Y_1 - Y_0) + P_1(Y_0 - W_1) > P_1(Y_1 - W_1),$$

where we use equation (22) for the first inequality and $P_0 < P_1$, and hence $(P_0 - P_1)(Y_1 - Y_0) > 0$ for the second inequality. The conclusion:

$$P_0(Y_1 - W_0) > P_1(Y_1 - W_1),$$

is inconsistent with the notion that $W_1 = \arg\max_W P(W)[Y_1 - W]$. This implies that $W_0 \geq W_1$. Finally, we must rule out the case in which $W(x)$ is constant for some x. For this we can use the standard Burdett and Mortensen (1998) argument that if $\widehat{F}(W)$ had mass points, offering a slightly higher wage would be a profitable deviation for firms. This implies that $W_0 > W_1$.■

A.2 Derivation of equation (7)

Totally differentiating (6) yields

$$V_x^E(x) = \frac{W_x(x)}{\rho+\delta+2\psi\lambda v S x}. \tag{23}$$

The solution to this differential equation is

$$V^E(x) = \int_0^x \frac{W_x(z)}{\rho+\delta+2\psi\lambda v S z} dz + C_0.$$

Integrating by parts yields

$$V^E(x) = \frac{W(x)}{\rho+\delta+2\psi\lambda v S x} - \frac{W(0)}{\rho+\delta} + 2\lambda v\psi \int_0^x \frac{W(z)}{(\rho+\delta+2\psi\lambda v S z)^2} dz + C_0. \tag{24}$$

Evaluating (24) at $x = 0$ gives an initial condition that can be used to solve for C_0:

$$C_0 = V^E(0) = \frac{W(0)}{\rho+\delta} + \frac{\delta}{\rho+\delta} V^U.$$

Substitution of this equation into (24) yields the desired expression.

A.3 Derivation of equation (18)

Rewriting equation (15) gives

$$W_x(x) + p(x)W(x) = s(x),$$

where

$$\begin{aligned} p(x) &\equiv -\frac{\psi\kappa v S}{1+\psi\kappa v S x}, \\ s(x) &\equiv -\frac{\psi\kappa v S\left(1-\frac{1}{2}\gamma x^2\right)}{1+\psi\kappa v S x}. \end{aligned}$$

The general form of the solution is as follow (see for example Kreyszig, 1993:31):

$$\begin{aligned} W(x) &= e^{-t(x)}\left[\int e^{t(x)} s(x) dx + c\right], \\ t(x) &\equiv \int p(x) dx. \end{aligned}$$

Substituting in $p(x)$ and $s(x)$ and solving the integrals gives:

$$\begin{aligned} W(x) = 1 + \frac{1}{2}\gamma x^2 - \gamma \frac{1+\psi\kappa v S x}{(\psi\kappa v S)^2} \log(1+\psi\kappa v S x) \\ + \frac{\gamma}{(\psi\kappa v S)^2} + \gamma \frac{x}{\psi\kappa v S} + c(1+\psi\kappa v S x). \end{aligned}$$

Solving c from the initial condition for $W(x)$, equation (12), yields equation (18).■

A.4 Proof of Proposition 4

Let

$$z(u) \equiv \psi \frac{1-u}{u}, \tag{25}$$

where z can be interpreted as the ratio of (effective) employed and unemployed job seekers in the search pool. Since z is a continuous and monotonically declining function of u with $z(1) = 0$ and $z(0) = \infty$, any positive value of z implies a unique value of u. Consider (11). First, we use the definition of $\mathrm{E}_G W = \int_0^{\overline{x}} W(x) g(x) dx$ and substitute equations (18) and (5) into this definition, and then we substitute the result into equations (11) and (13). Solving the integrals, and substituting in $\kappa S v \overline{x}$ for $(1-u)/u$, see equation (3), gives the equilibrium values of z and $\overline{x}$ as a solution to the following system of equations:

$$\frac{\psi z^2}{z^2 + (1-\psi)\log(1+z)\left[\log(1+z) - 2z\right]} = B^* \overline{x}^2, \tag{26}$$

$$B^* \equiv \frac{\frac{1}{2}\gamma}{1-B},$$

$$Q(z,\psi) \equiv 2^{\frac{1}{2}+\xi} \psi^{\frac{1}{2}+\xi+\eta} \left(\frac{1+z}{\psi+z}\right)^{\xi+\eta-1} \times$$

$$\frac{\left\{\log(1+z)\left[1+z-\frac{1}{2}\log(1+z)\right] - z\right\}^{\xi}}{\left\{z^2 + (1-\psi)\log(1+z)\left[\log(1+z) - 2z\right]\right\}^{\frac{1}{2}+\xi}} = K^* \left(\frac{1-B}{K}\right)^{1-\xi}. \tag{27}$$

Next, we must prove that the equilibrium condition (27) has an interior solution for $z > 0$. We start our proof for $\xi = 1$. Sequential use of l'Hopital's rule gives

$$\lim_{z\to 0} \frac{\left\{\log(1+z)\left[1+z-\frac{1}{2}\log(1+z)\right] - z^2\right\}^2}{\left\{z^2 + (1-\psi)\log(1+z)\left[\log(1+z) - 2z\right]\right\}^3} = \frac{1}{9\psi^3},$$

$$\lim_{z\to \infty} \frac{\left\{\log(1+z)\left[1+z-\frac{1}{2}\log(1+z)\right] - z^2\right\}^2}{\left\{z^2 + (1-\psi)\log(1+z)\left[\log(1+z) - 2z\right]\right\}^3} = 0.$$

Hence $\lim_{z\to 0} Q(z,\psi) = \frac{2}{3}\sqrt{2}$ and $\lim_{z\to\infty} Q(z,\psi) = 0$. This implies that if

$$\frac{2}{3}\sqrt{2} \geq K^*,$$

then equation (27) has at least one root, proving part (ii) of the proposition. In addition, for $\xi = 1$, $Q(z,\psi)$ can be shown to be decreasing in z. Hence, $\frac{2}{3}\sqrt{2}$ is the maximum of this function and there is no interior solution if $\frac{2}{3}\sqrt{2} < K^* \left(\frac{1-B}{K}\right)^{1-\xi}$. For $\xi < 1$, we can use the result for $\xi = 1$ to obtain

$$\lim_{z\to 0} \frac{\left\{\log(1+z)\left[1+z-\frac{1}{2}\log(1+z)\right]-z^2\right\}^{\xi}}{\left\{z^2+(1-\psi)\log(1+z)\left[\log(1+z)-2z\right]\right\}^{\frac{1}{2}+\xi}}$$
$$= \frac{1}{9\psi^3}\left(\lim_{z\to 0}\frac{z^2+(1-\psi)\log(1+z)\left[\log(1+z)-2z\right]}{\log(1+z)\left[1+z-\frac{1}{2}\log(1+z)\right]-z^2}\right)^{1-\xi} = \infty,$$

using l'Hopital's rule for the last equality. Hence $\lim_{z\to 0} Q(z,\psi) = \infty$. In a similar fashion it can be shown that $\lim_{z\to\infty} Q(z,\psi) = 0$. This proves part (i) of the proposition.

For the case that $\psi = 1$, equation (27) simplifies to

$$2^{\frac{1}{2}+\xi}\left(\frac{u}{1-u}\right)^{1+\xi}\left(-\frac{1+\frac{1}{2}u\log u}{1-u}\log u - 1\right)^{\xi} = K^*\left(\frac{1-B}{K}\right)^{1-\xi}. \quad (28)$$

We must prove that the left-hand side of equation (28) is monotonic in u. Its first-order derivative has the same sign as

$$(1-u)^2\log(u) - \left(2+\frac{1}{\xi}\right)\left[\left(1+\frac{1}{2}u\log u\right)\log u + 1 - u\right]. \quad (29)$$

One can check numerically that this function is positive for $\xi = 1$. For lower values of ξ, it is larger, since

$$\left(1+\frac{1}{2}u\log u\right)\log u + 1 - u < 0.$$

■

A.5 Comparative statics for $\beta = 0, \psi = 1$

Total differentiation of the equilibrium condition (28) results in the derivatives of u with respect to the parameters of the model. Since the left-hand side of this equation is increasing in u, the sign of the first-order derivative depends only on these parameters. In addition, we have that $\overline{x} = \sqrt{1/B^*}$, and using this we can directly find the results for $\overline{x}$. Finally, we us equation (3) and $S = v^{\xi-1}$ to solve for v; taking derivatives and substituting in $\frac{du}{d\kappa}$ yields:

$$\frac{dv}{d\kappa} = 2^{\frac{1}{2}+\xi}\left(-\frac{1+\frac{1}{2}u\log u}{1-u}\log u - 1\right)^{\xi-1}\frac{\log u\left(2u+1-u^2+u\log u\right)+2(1-u)}{\xi\kappa^{1+\frac{1}{\xi}}\overline{x}^{\frac{1}{\xi}}u^{\frac{1}{\xi}-\xi}(1-u)^{3-\frac{1}{\xi}+\xi}f'(u)}.$$

The sign of this relationship depends on the sign of

$$\log u \left(2u + 1 - u^2 + u \log u\right) + 2\left(1 - u\right).$$

This function is negative for small values of u and positive for larger values. The function has a single root at $u \approx 27\%$. Hence, there is a non-monotonic relationship between v and κ.

A.6 Derivation of equations (17) and (19)

Equation (16) is equivalent to

$$\arg\max_{W} \left(\left\{ u + \psi\left(1-u\right)\left[1 - \widehat{G}\left(W\right)\right] \right\} \frac{Y\left(x\right) - W}{\rho + \delta + 2\psi\lambda v \overline{x} S \widehat{F}(W)} \right)$$

The FOC with respect to W reads

$$0 = -\frac{\psi\left(1-u\right) G_x / W_x}{u + \psi\left(1-u\right)\left[1 - \widehat{G}\left(W\right)\right]} - \frac{2\psi\lambda v \overline{x} S F_x / W_x}{\rho + \delta + 2\psi\lambda S v \overline{x} \widehat{F}(W)} - \frac{1}{Y\left(x\right) - W},$$

where we use $\widehat{G}_W = G_x/W_x$ and $\widehat{F}_W = F_x/W_x$. Using $F_x = 1/\overline{x}$ and some rearrangement yields:

$$W_x = -\left(\frac{\psi\left(1-u\right) G_x}{u + \psi\left(1-u\right)\left[1 - \widehat{G}\left(W\right)\right]} + \frac{\psi\kappa S v}{1 + \psi\kappa S v x} \right) \left[Y\left(x\right) - W\right]. \qquad (30)$$

Use (4) and its derivative with respect to x to write,

$$\frac{\psi\left(1-u\right) G_x}{u + \psi\left(1-u\right)\left[1 - \widehat{G}\left(W\right)\right]} = \frac{\psi\kappa S v}{1 + \psi\kappa S v x},$$

and substitute this back in (30) to get

$$W_x\left(x\right) = -\frac{2\psi\kappa v S\left(Y(x) - W\left(x\right)\right)}{1 + \psi\kappa v S x}.$$

This equation is almost identical to equation (15). For the solution of this type of differential equation we therefore refer to appendix A.3. Applying this solution and solving the initial condition yields equation (19).■

A.7 Proof of Proposition 4 for wage setting with commitment

Use the definition of $\mathrm{E}_G W = \int_0^{\overline{x}} W(x)g(x)dx$ and substitute equations (19) and (5) into this definition, and substitute the result into equations (11) and (13). Solving for the integrals reveals that z and $\overline{x}$ (for the model of wage setting with commitment with $0 < \psi \leq 1$) are the solution to the following system of equations

$$z^2 \left\{ \frac{1-\psi}{\psi} \left[\frac{3+z}{1+z} z^2 + 6\log(1+z) - 6z \right] + z^2 \right\}^{-1} = B^* \overline{x}^2,$$

$$P(z,\psi) \equiv 2^{\frac{1}{2}+\xi} \psi^{\xi} \left(\frac{1+z}{\psi+z} \right)^{\xi+\eta-1} \times$$

$$\frac{\left\{-2\log(1+z) + 3z - \frac{1+2z}{1+z} z\right\}^{\xi}}{\left\{ \frac{1-\psi}{\psi} \left[\frac{3+z}{1+z} z^2 + 6\log(1+z) - 6z \right] + z^2 \right\}^{\frac{1}{2}+\xi}} = K^* \left(\frac{1-B}{K} \right)^{1-\xi}. \tag{31}$$

Next, we must prove that the equilibrium condition in (31) has an interior solution for $z > 0$. We start our proof for $\xi = 1$. By sequential use of l'Hopital's rule,

$$\lim_{z \to 0} \frac{\left\{-2\log(1+z) + 3z - \frac{2z+1}{1+z} z\right\}^2}{\left\{ \frac{1-\psi}{\psi} \left\{ \frac{z+3}{z+1} z^2 + 6\log(1+z) - 6z \right\} + z^2 \right\}^3} = \frac{1}{9}.$$

Hence $\lim_{z\to 0} P(z,\psi) = \frac{2}{3}\sqrt{2}$ and $\lim_{z\to\infty} P(z,\psi) = 0$. This implies that if:

$$\frac{2}{3}\sqrt{2} \geq K^*,$$

then equation (31) has at least one root, proving part (ii) of the proposition. We can also prove part (i) of the proposition by using this result and making the same arguments as in Proposition 4.

For the final part of the proof we consider the special case $\psi = 1$. Then, equation (31) simplifies to

$$2^{\frac{1}{2}+\xi} \left(\frac{u}{1-u} \right)^{1+\xi} \left(\frac{1-u^2+2u\log u}{1-u} \right)^{\xi} = K^* \left(\frac{1-B}{K} \right)^{1-\xi}. \tag{32}$$

Note that the derivative of this equation has the same sign as:

$$-(1-u)^3 + \left(2 + \frac{1}{\xi}\right) \left(1 - u^2 + 2u\log u\right).$$

One can check numerically that this function is positive for $\xi = 1$. For lower values of ξ, it is larger, since:

$$1 - u^2 + 2u\log u > 0.$$

■

A.8 Comparative statics for wage setting with commitment and $\psi = 1$

The derivations are similar to those obtained in Appendix A.5 using equation (32) instead of equation (27). The equation for the partial derivative of v with respect to κ equals

$$\frac{dv}{d\kappa}=-2\sqrt{2}\xi\left(2\xi u^2\frac{1-u^2+2u\log u}{(1-u)^3}\right)^{\xi-1}\frac{-5u^2+u^3+4u\log u+1+3u}{\kappa^{\frac{1}{\xi}+1}\overline{x}^{\frac{1}{\xi}}u^{\frac{1}{\xi}-2+\xi}(1-u)^{5-\frac{1}{\xi}-\xi}f'(u)}$$

The sign of this relationship depends on the sign of

$$-5u^2+u^3+4u\log u+1+3u.$$

This function is negative for small values of u and positive for larger values. The function has a single root at $u \approx 39\%$. Hence, there is a non-monotonic relationship between v and κ.

A.9 Proof of Proposition 6

Substitution of the definition of $\mathrm{E}_G(Y)$ together with equation (3) in (20) yields an expression for Ω as a function of the acceptance rule $\overline{x}$ and unemployment u. In addition, when $\psi = 1$, we have $\overline{x} = \sqrt{1/B^*}$; hence, this expression can be rewritten as

$$\Omega = 1-(1-B)\left[2\frac{u}{(1-u)^2}(u\log u-u+1)+\left(\frac{1}{2}\sqrt{2}\frac{1-u}{u}K^*\left(\frac{1-B}{K}\right)^{1-\xi}\right)^{1/\xi}\right]. \quad (33)$$

The first-order condition for maximization of this expression is:

$$2^{\frac{1}{2}+\xi}\left(\xi\frac{1-{u^*}^2+2u^*\log u^*}{1-u^*}\right)^{\xi}\left(\frac{u^*}{1-u^*}\right)^{1+\xi}=K^*\left(\frac{1-B}{K}\right)^{1-\xi}. \quad (34)$$

Figure 4 compares u for the market and the social planner for $\xi = 1$. The upper curve represents the decentralized equilibrium, (28); the lower curve is the unemployment rate preferred by the social planner from equation (34). The vertical axis gives the value of the left-hand side of those equations. The right-hand side of both equations is equal to $K^*\frac{1-B}{K}$, which does not depend on u and can therefore be represented by a horizontal line. The intersections of this horizontal line with the two other curves give the market and the planner's value for u. For all values $K^*\frac{1-B}{K}$, unemployment is too low and vacancies are too high in the market under monopsony.■

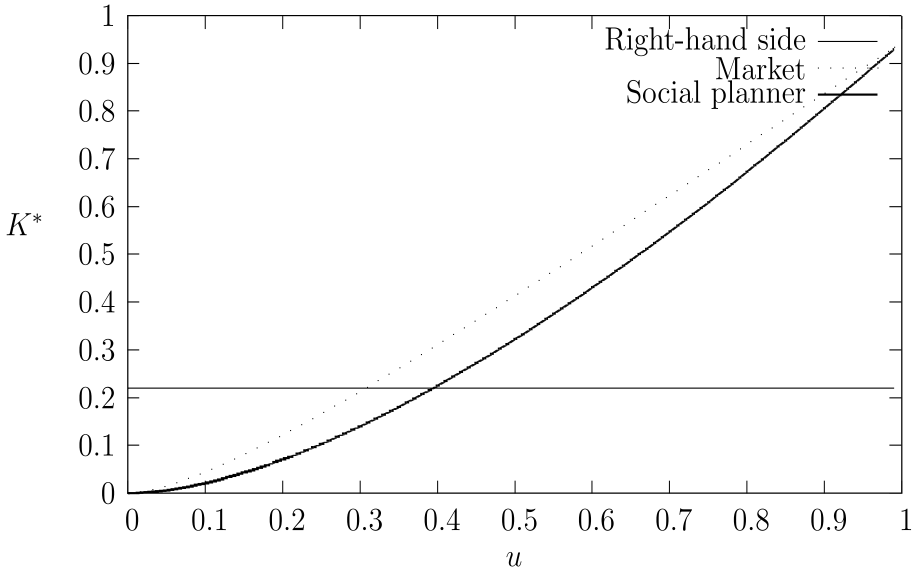

Figure 4: Planner's and decentralized unemployment for $\psi = \xi = 1$

A.10 Proof of Proposition 7

The second-best level of unemployment insurance maximizes $\Omega^- = \Omega - uB$ with respect to B (see equation (33) for Ω, subject to equation (28)). Define $f(u)$ as the left-hand side of equation (28) for $\xi = 1$:

$$f(u) \equiv -2\sqrt{2}\left(\frac{u}{1-u}\right)^2 \left(\frac{1+\frac{1}{2}u\log u}{1-u}\log u + 1\right).$$

Using this definition, the total derivative of equation (28) reads

$$\frac{du}{dB} = \frac{3}{2}\frac{1}{1-B}\frac{f(u)}{f'(u)}.$$

Hence, the total derivative of Ω^- with respect to B reads

$$\begin{aligned}\frac{d\Omega^-}{dB} &= \frac{3u}{(1-u)^2}\frac{f(u)}{f'(u)}\left[\frac{1+u}{u}\log(u) + \frac{\log^2 u}{1-u} + \frac{1-u}{u}\right] \\ &\quad - \frac{3}{2}\frac{f(u)}{f'(u)}\frac{B}{1-B} - u - 2\left[\left(\frac{u}{1-u}\right)^2 \log u \left(\frac{1-u}{u} + \frac{1}{2}u\right)\right]\end{aligned} \tag{35}$$

We should prove $d\Omega^-/dB$ to be positive for $B = 0$. One can show numerically that this is the case for any value of u between zero and one.■

A.11 Free-entry condition under Bertrand competition

In Postel-Vinay and Robin (2002), wages are determined by Bertrand competition between poaching and incumbent firms. Let $W(x,z)$ be the wage of a worker who used to be employed at z and who is currently employed at a firm at distance x from her optimal job.[17]. From Postel-Vinay and Robin's (2002) equation (3) it follows that:

$$W(x,z) = Y(z) - \frac{2\lambda v}{\rho+\delta}\int_x^z qY'(q)dq. \tag{36}$$

Note that $W(x,z)$ is also the wage of a worker with an x match whose highest offer so far was from a firm at distance z. The Bellman equation for a firm who employs a worker at distance x who previously was employed at distance z ($V^J(x,z)$) is given by:

$$V^J(x,z) = \frac{Y(x) - W(x,z) + 2\lambda vx \int_x^z V^J(x,q)\,dq}{\rho+\delta+2\lambda vz} \tag{37}$$

Note that $V^J(x,z)$ is equal to the value of a filled job for a firm at distance x that counter-offered a firm with productivity z. Also note that this implies that firms only have to consider the offer of the firm at the shortest distance so far. Equation (37) is an implicit equation in $V^J(x,z)$. In order to get an explicit expression, note that the partial derivative of $V^J(x,z)$ equals:

$$\frac{\partial V^J(x,z)}{\partial z} = -\frac{\frac{\partial W(x,z)}{\partial z}}{\rho+\delta+2\lambda vz}, \tag{38}$$

where

$$\frac{\partial W(x,z)}{\partial z} = Y'(z)\left(1+\frac{2\lambda v}{\rho+\delta}z\right).$$

Substitution into (38) gives the differential equation:

$$\frac{\partial V^J(x,z)}{\partial z} = -\frac{1}{\rho+\delta}Y'(z)$$

The initial condition follows from the fact that Bertrand competition implies that $V^J(x,x) = 0$. Solving the differential equation then gives:

$$V^J(x,z) = \frac{Y(x)-Y(z)}{\rho+\delta}. \tag{39}$$

[17]For a worker who comes out of unemployment, x_1 equals $\overline{x}$. Hence, the reservation wage is equal to $W(x_0,\overline{x})$.

The free entry can then be written as

$$\kappa u\int\limits_0^{\overline{x}}\left(1-\frac{1}{2}\gamma x^2-B\right)dx+\kappa\left(1-u\right)\int\limits_0^{\overline{x}}g(z)\int\limits_0^{z}\frac{1}{2}\gamma\left(z^2-x^2\right)dxdz-K=0.$$

Plugging (5) and (12) into this condition, solving the integrals and using (3) gives (after some algebra)

$$2\sqrt{2}\left(\frac{u}{1-u}\right)^2\left(-\frac{u}{(1-u)}\log u-1+\frac{1}{2}\frac{1-u}{u}\right)=K^*.$$

Figure 5 shows the left-hand sides of this equation for different values of u together with those for the two wage schemes without matching outside offers. For given K^*, unemployment is lowest in the PR model, implying that there is excessive vacancy creation. Below, we give a direct proof that workers receive less than their shadow value in this wage mechanism. The shadow values of an unemployed and employed worker are given by (6) and (8) where E_GW is replaced by E_GY, V^U is replaced by V^{U^*} and $V^E(x)$ is replaced by $V^{E^*}(x)$. It implies that the social value of a match of a firm at distance x that meets a worker at distance $z<x$ equals $V^{E^*}(x)-V^{E^*}(z)$. Using (6), we obtain

$$V^{E^*}(x)-V^{E^*}(z)=\frac{Y(x)}{\rho+\delta+2\lambda vx}-\frac{Y(z)}{\rho+\delta+2\lambda vz}-2\lambda v\int_x^z\frac{Y(t)}{(\rho+\delta+2\lambda vz)^2}dt.$$

Taking derivatives of the left- and right-hand sides, we obtain

$$\frac{\partial\left(V^{E^*}(x)-V^{E^*}(z)\right)}{\partial z}=\frac{Y'(z)}{\rho+\delta+2\lambda vz}.$$

Now consider a wage schedule $W^*(x,z)$ in which the firm is paid exactly the contribution of the match. This wage schedule is by definition efficient. ¿From the above we know that the first-order derivative of the value of a filled job for the firm equals

$$\frac{\partial V^J\left(x,z\right)}{\partial z}=-\frac{\frac{\partial W^*(x,z)}{\partial z}}{\rho+\delta+2\lambda vz}.$$

In order to make V^J equal to $V^{E^*}(x)-V^{E^*}(z)$, the first-order derivatives need to match and hence $\frac{\partial W^*(x,z)}{\partial z}$ must be equal to $Y'(z)$, since $W^*\left(x,z\right)=Y(z)$. From (36) it follows that $W\left(x,z\right)<W^*\left(x,z\right)$.■

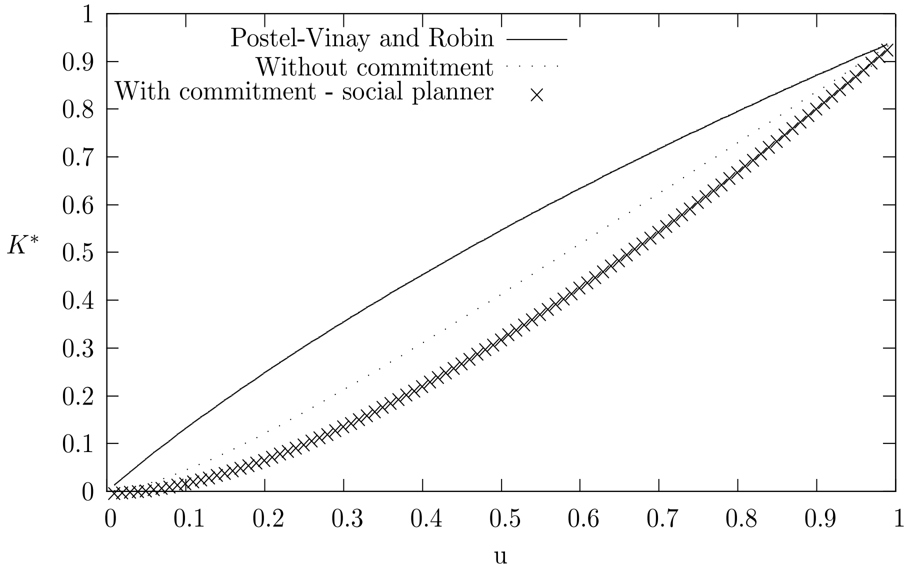

Figure 5: Unemployment for different wage schedules.